Enhancement of the ensemble-averaged coupling between defects in random environments

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We theoretically and experimentally explore the coupling behavior of defect pairs immersed in amorphous host lattices. Our observations in photonic lattice arrangements clearly demonstrate how such disordered environments can systematically accelerate the hopping dynamics between localized defect states. © 2014 Optical Society of America

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Photonic band gaps hold great promise to efficiently confine, conduct and steer light in integrated structures. Going beyond conventional index-guiding arrangements, photonic band gap systems can achieve a remarkable degree of control over optical wave packets, similar to electrons in semiconductor heterostructures [1,2]. Consequently, such notions have been at the focus of intense research over the past twenty-five years. Going beyond the analogy to solid state physics, band gap structures have been successfully employed in a number of distinctly optical applications of high technological relevance, ranging from hollow-core photonic crystal fibers for channeling intense laser beams [3] and highly nonlinear microstructured fibers for super-continuum generation [4] to omnidirectional mirrors [5], efficient optical filters [6], and switches [7], to name just a few. Finally, photonic band gap structures have been instrumental to a variety of studies on fundamental concepts, such as Anderson localization [8,9] and discrete solitons [10–13] in periodic media. The underlying mechanism enabling most of these effects, namely the capability to localize, channel, and manipulate the electromagnetic energy in defect modes, is clearly one of the most important aspects of photonic band gap structures.

Interestingly, band gaps are by no means an exclusive feature of periodic structures. Contrary to conventional wisdom, amorphous systems are indeed capable of exhibiting well-defined gaps in their spectrum of eigenstates—despite the fact that the absence of any long-range order prohibits Bragg scattering [14–16]. Furthermore, photonic band gaps have also been observed in amorphous systems in the microwave regime [17]. Only recently, the concept of amorphous band gaps was introduced to the field of optics, and subsequently investigated in liquid-like arrangements of waveguides [18]. In these experiments, the guiding of light by means of an isolated defect state residing deep inside the gap could be demonstrated. Yet, to this date, no systematic investigation of the fundamental nature of amorphous band gap defect states has been undertaken to our knowledge. While one might certainly expect them to inherit some characteristics of their counterparts in periodic settings, the stochastic properties of an amorphous environment will undoubtedly have a profound influence on the nature of the interaction between such defect states. The question naturally arises as to how individual defects may interact with one another, and over which distances this interaction can take place. In this Letter, we present a comprehensive study of coupled defect states residing in the band gap of disordered photonic lattices. We experimentally observe and theoretically describe a distinct enhancement of long-range coupling by virtue of the amorphous environment.

The transfer of optical power between localized modes of any system can formally be described in terms of coupling between their modal amplitudes, e.g., a_1, a_2 [19]. The resulting equation

$$-i\frac{d}{dz}\begin{pmatrix}a_1\\a_2\end{pmatrix}=\mathscr{H}\begin{pmatrix}a_1\\a_2\end{pmatrix}, \tag{1}$$

governs the dynamics of light propagating in this arrangement along the longitudinal coordinate z, as mediated by the Hamiltonian $\mathscr{H}=\left(\begin{smallmatrix}\beta & c\\ c & 0\end{smallmatrix}\right)$. Here, β represents the detuning between the interacting defect states, and the coefficient c indicates the strength of coupling between their respective wave functions. A tuned system with identical defects is characterized by $\beta=0$, whereas $\beta\neq 0$ corresponds to defect waveguides of different propagation constants. With the two eigenvalues $\lambda_\pm=(\beta/2)\pm((1/4)\beta^2+c^2)^{1/2}$ of $\mathscr{H}$, one can define the quantity $\kappa=(1/2)|\lambda_+-\lambda_-|$ to describe the effective "hopping rate," or beating frequency, between the two states. In a fully deterministic system, e.g., a directional coupler, or two defect states in a photonic crystal, β is constant, and the effective hopping rate is given by [20]

$$\kappa_{\mathrm{det}}=\sqrt{\frac{\beta^2}{4}+c^2}. \tag{2}$$

On the other hand, the influence of an amorphous environment manifests itself as a random detuning β. In our study we model this stochastic quantity as an ensemble obeying a uniform distribution around the tuned regime, i.e., $\beta_{\mathrm{sto}} \in [-s/2, s/2]$, where each individual β_{sto} corresponds to a specific realization of the background lattice [see e.g., Fig. 1(a)].

On a fundamental level, this problem is related to "random matrices," a field of research pioneered by Wigner [21]. Using the eigenvectors of $\mathscr{H}$, one can compute an expectation value for the stochastic hopping between the defect states, which takes the form

$$\langle \kappa_{\mathrm{sto}} \rangle = \frac{1}{2}\sqrt{\left(\frac{s}{4}\right)^2 + c^2} + \left[\frac{2c^2}{s}\operatorname{arcsinh}\left(\frac{s}{4c}\right)\right]. \quad (3)$$

Whereas the first term in this expression is reminiscent of the deterministic case [compare to Eq. (2)], the second term is of entirely stochastic origin and may cause significant deviations from the ordered regime. Surprisingly, it is strictly positive: any amorphous environment necessarily accelerates the hopping between enclosed defect states. Note that this enhancement is inherently linked to the ratio s/c. Naturally, the expectation value converges to the deterministic case in the limit of vanishing disorder:

$$\lim_{s\to 0} \langle \kappa_{\mathrm{sto}} \rangle = c. \quad (4)$$

On the other hand, for a given fixed value $s = \text{const.}$, the strength of this unusual influence of randomness depends on the coupling, as shown in Fig. 1(b) in comparison to the behavior in tuned and deterministically detuned couplers. Taking into account the relation $c(d)$ between coupling c and spatial separation d in a physical system, this means that the overall hopping is composed of a constant effective detuning and distance-dependent term. Note that for any nonzero detuning, be it deterministic or stochastic in nature, κ retains a finite value for zero coupling. This of course does not translate to an actual power transfer over infinite distances. Instead, the intensity "beating contrast"

$$K = 1 - \frac{\min_z |a_1(z)|^2}{\max_z |a_1(z)|^2} = \frac{c^2}{\beta^2 + c^2} \quad (5)$$

between the channels of any deterministically detuned coupler ($\beta \neq 0$) vanishes as c approaches zero.

In order to observe the consequences of these findings, we employed the femtosecond laser inscription technique [22,23] to fabricate various amorphous photonic lattice configurations in fused silica glass. Each realization was comprised of 250 waveguides and featured a propagation length of 25 mm. Owing to the specifics of the fabrication method, the waveguides exhibited elliptical cross-sections (11 μm × 3 μm) with a refractive index contrast of $\Delta n = 9 \cdot 10^{-4}$ on top of the bulk refractive index $n_0 = 1.45$. Within each lattice, we embedded a coupled pair of identical defect waveguides fabricated at slightly higher writing velocities. The resulting index contrast of $\Delta n_d = 5 \cdot 10^{-4}$ yielded a negative detuning with respect to the discrete environment, thereby placing the respective propagation constants within the band gap of the structure. The surrounding lattice sites were arranged

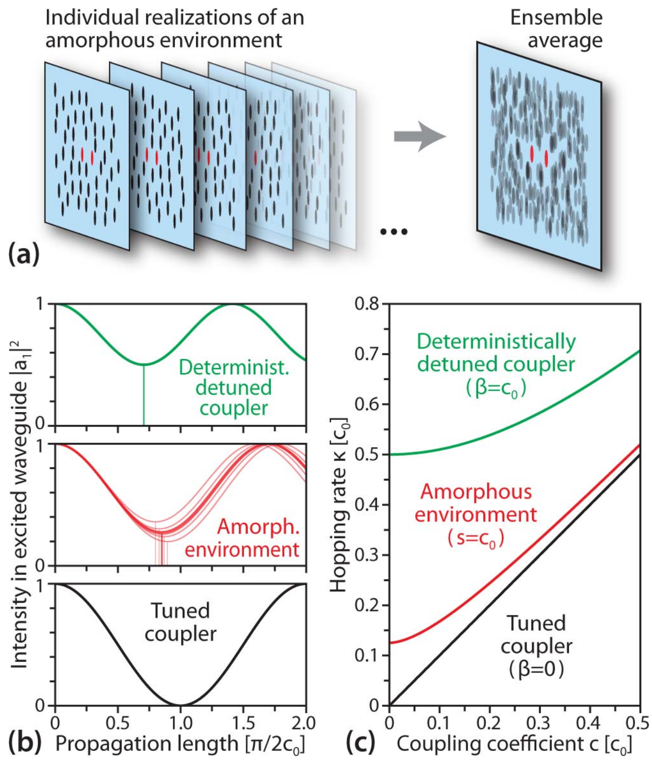

Fig. 1. (a) Schematic of two defect waveguides (red) embedded within various realizations of an amorphous environment (black). The propagation constants associated with the defects (red) reside within the photonic band gap of the structure. (b) Periodic intensity transfer between two coupled waveguides. Bottom: tuned coupler; top: deterministically detuned coupler ($\beta = c_0$); and center: coupler in amorphous environment, displaying a stochastic detuning, as schematically indicated by the traces of individual realizations. (c) Dependence of the hopping rate κ associated with these examples on the coupling coefficient c. For all values of c, the stochastic parameter of the amorphous environment was fixed ($s = c_0$). All quantities are normalized with respect to the scale of an arbitrary coupling c_0.

according to a liquid-like model to ensure the absence of any long-range order and the associated Bragg reflection [18]. As a reference representing the deterministic case, similar pairs of identical waveguides ($\Delta n = 5 \cdot 10^{-4}$) were fabricated without a background lattice.

Figure 2(a) shows a representative example micrograph of an amorphous lattice. As can be seen from the output facet of the sample [Fig. 2(b)], light launched into one of the defect waveguides is gradually transferred between the defect states upon propagation in accordance with the stochastic perturbation brought about by the random positioning of the background lattice. In total, we characterized the hopping between the defect waveguides by taking the ensemble average over 11 respective realizations for each of the 9 different values of the defect spacing. The evolution dynamics were directly observed by means of waveguide fluorescence microscopy [24] at a wavelength of 633 nm [see Figs. 2(c) and 2(d)]. This approach allowed us to precisely retrieve the beating length L_B, without any ambiguities due to the periodic propagation pattern. The value of the hopping rate was then calculated with $\kappa = \pi/2L_B$ and compared to the deterministic reference. In full agreement with the

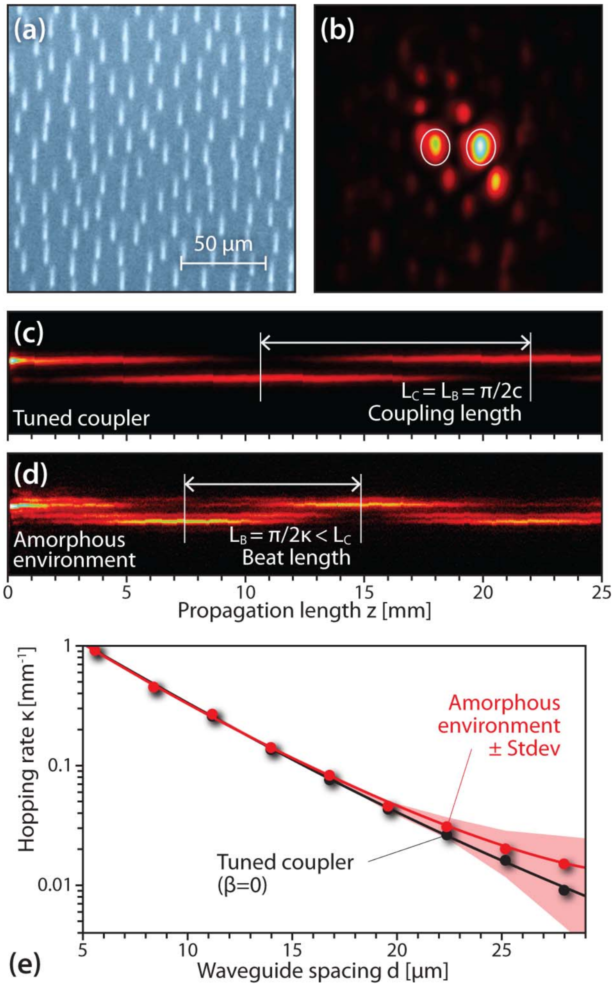

Fig. 2. (a) Micrograph of a typical realization of an amorphous photonic lattice. The individual waveguides have elliptical cross-sections of 3 μm × 11 μm and their positions distributed according to a liquid-like model. (b) Output pattern obtained from a coupled defect pair when one of the channels is selectively excited. To resolve ambiguities due to the periodic coupling dynamics, we directly observed the propagation by means of waveguide fluorescence microscopy. Shown are examples of evolution patterns for (c) a tuned coupler and (d) a defect pair embedded within an amorphous background lattice, each with a spacing of $d = 14$ μm. (e) Experimentally retrieved hopping rates κ for the amorphous and the deterministic (tuned) system. The random detuning of the defect guides by the amorphous environment clearly results in an accelerated hopping for large spacings.

theoretical predictions the experimental data show that with spacings greater than 16 μm, the hopping is notably and systematically enhanced in the presence of an amorphous environment [see Fig. 2(e)].

The statistical character of our observation gives rise to an increasing standard deviation of the hopping with spacing, as shown in Fig. 3(a). Nevertheless, we detected a clear signature of the predicted hopping enhancement in our experiments. Figure 3(b) illustrates the increasing deviation between the mean-square and square-mean hopping for large spacings, which serves as a measure for the probability distribution of the random detuning. This provides direct experimental proof of the nondeterministic acceleration in the hopping dynamics between

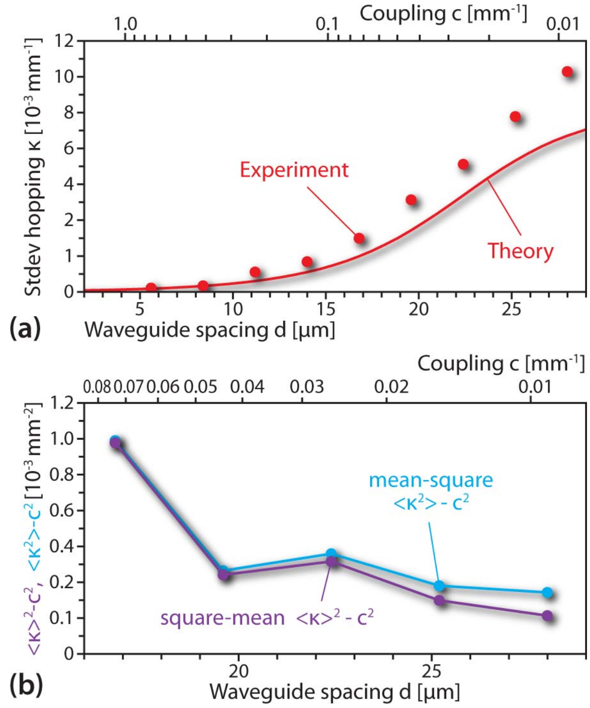

Fig. 3. (a) Experimentally extracted standard deviation of the hopping constant, compared to the theoretical prediction. (b) Experimental results for the mean-square and square-mean hopping coefficient, showing stochastic behavior: $sm < ms$ at large defect spacing. The ratio between the two is a function of the probability distribution of the random detuning.

the defect waveguides mediated by the amorphous environment.

In conclusion, we have theoretically and experimentally explored the coupling dynamics between defect states embedded within amorphous lattice environments. Our results clearly demonstrate that in the ensemble average, the presence of a random environment tends to systematically increase the hopping rate between localized defect waveguides. A similar behavior can be expected in randomly detuned systems of non-identical defects. Due to the general nature of the underlying mathematical framework, our results are applicable to the dynamics of coupled defects in any amorphous systems, in optics and beyond.

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